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# Adaptive control and stability analysis of nonlinear crane systems with perturbation

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#### Abstract

This paper presents an adaptive control approach using a model matching technique for 3-DOF nonlinear crane systems. The proposed control is linearly composed of two control frameworks: nominal PD control and corrective control. A nonlinear crane model is approximated by means of feedback linearization to design nominal PD control avoiding perturbation. We propose corrective control to compensate system error feasibly occurring due to perturbation, which is derived by using Lyapunov stability theory with bound of perturbation. Additionally, we achieve stability analysis for the proposed crane control system and analytically derive sufficient stability condition with respect to its perturbation. Numerical simulation is accomplished to evaluate our proposed control and demonstrate its reliability and superiority compared to traditional control method.

Keywords: Crane control; Model matching; Perturbation; Lyapunov theory; Feedback linearization

## 1. Introduction

A crane is an important system industrially for carrying a heavy object to a desired position within a given time interval. Recently, more efficient cranes have been investigated and, correspondingly, advanced control approaches for such systems are being addressed.

Recent addressed cranes are mostly involved with complicated nonlinear systems, but for simplicity, most engineers usually approximate or linearize the model in control design procedures to apply linear system theory. Feedback linearization is well used for such procedure by which the crane model is linearly approximated and then a nominal control framework such as PD control, state-feedback control, etc. are employed [1-5]. More recently, enhanced control has been addressed in [6, 7], and [8] using passivity theory [9] and intelligent algorithm [10]. Until now, most authors have avoided the perturbation problem in crane systems, which is possibly yielded due to modeling error, change of system environment, and uncertainty in practice. In reality, information about system perturbation is partially known such as its upper or lower bound, which is a significant factor for system stability. Moreover, this information is used to establish robust control strategy for perturbed crane systems.

This paper presents a nonlinear crane control system against non-vanishing perturbation. The control framework is linearly composed of two control configurations: PD control and corrective control. We first design PD control by using a nominal crane model without perturbation, which is linearly approximated by means of feedback linearization. Then, we derive a corrective control law for the perturbed crane model via Lyapunov stability theory. We consider the change of payload mass as crane perturbation, which is realistic in practice. Corrective control is an auxiliary input to compensate system error due to perturbation nature. We accomplish stability analysis for a perturbed crane system embedded with the

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proposed control method through Lyapunov perturbation stability theory. We assure that the proposed control methodology is referred as a design guideline for prospective crane control systems. Computer simulation is carried out for evaluating our control method and demonstrating its superiority compared to the traditional control method.

The remainder of this paper is organized as follows: In section 2 we describe a 3-DOF crane system. In section 3 the proposed control design is derived for a perturbed crane system. Stability analysis of the control system is carried out in section 4. Numerical simulation is presented in section 5. Finally, the conclusion and future work are given in section 6.

## 2. Nonlinear crane model

We consider a 3-DOF overhead crane system in this paper, shown in Fig. 1. The dynamic equation of this system is given by

$$\mathbf{M}\begin{bmatrix} \ddot{x}\\ \ddot{y}\\ \ddot{y}\\ \ddot{\theta}\\ \ddot{\phi} \end{bmatrix} + \mathbf{V}\begin{bmatrix} \dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{\phi} \end{bmatrix} + G = \begin{bmatrix} f_x\\ f_y\\ 0\\ 0 \end{bmatrix}$$
(1)

where  $f_x$  and  $f_y$  are the input forces acting on the cart and the rail, respectively; state *x* and *y* are the positions along two axis, and  $\theta$  is the payload angle with respect to vertical direction, and  $\phi$  is the projection of the payload angle. The corresponding matrices in (1) are given by

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$
(2)

with

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$$m_{11} = m_p + m_r + m_c$$
  

$$m_{12} = m_{21} = m_{34} = m_{43} = 0$$
  

$$m_{13} = m_p L \cos(\theta) \sin(\phi)$$
  

$$m_{14} = m_p L \sin(\theta) \cos(\phi)$$
  

$$m_{22} = m_p + m_c$$
  

$$m_{23} = m_p L \cos(\theta) \cos(\phi)$$
  

$$m_{24} = -m_p L \sin(\theta) \sin(\phi)$$

$$m_{31} = m_p L \sin(\theta) \sin(\phi)$$
  

$$m_{32} = m_p L \cos(\theta) \cos(\phi)$$
  

$$m_{33} = m_p L^2 + I$$
  

$$m_{41} = m_p \sin(\theta) \cos(\phi)$$
  

$$m_{42} = -m_p L \sin(\theta) \sin(\phi)$$
  

$$m_{44} = m_p L^2 \sin^2(\theta) + I$$

where  $m_p$ ,  $m_c$ , and  $m_r$  are mass of a payload, a cart, and a rail,

$$V = \begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} \\ v_{21} & v_{22} & v_{23} & v_{24} \\ v_{31} & v_{32} & v_{33} & v_{34} \\ v_{41} & v_{42} & v_{43} & v_{44} \end{bmatrix}$$
(3)

with

$$\begin{split} v_{11} &= v_{12} = v_{21} = v_{22} = v_{31} = v_{32} = v_{41} = v_{42} = 0 \\ v_{13} &= -m_p L \sin(\theta) \sin(\phi) \dot{\theta} + m_p L \cos(\theta) \cos(\phi) \dot{\phi} \\ v_{14} &= m_p L \cos(\theta) \cos(\phi) \dot{\theta} - m_p L \sin(\theta) \sin(\phi) \dot{\phi} \\ v_{23} &= -m_p L \sin(\theta) \cos(\phi) \dot{\theta} - m_p L \cos(\theta) \sin(\phi) \dot{\phi} \\ v_{24} &= -m_p L \cos(\theta) \sin(\phi) \dot{\theta} - m_p L \sin(\theta) \cos(\phi) \dot{\phi} \\ v_{34} &= -m_p L^2 \sin(\theta) \cos(\phi) \dot{\phi} \\ v_{43} &= m_p L^2 \sin(\theta) \cos(\phi) \dot{\phi} \\ v_{44} &= m_p L^2 \sin(\theta) \sin(\phi) \dot{\theta} \end{split}$$

and

$$G = \begin{bmatrix} 0 & 0 & m_p gL\sin(\theta) & 0 \end{bmatrix}^T$$
(4)



Fig. 1. A crane system model.

We make the following assumptions about the system model: First, the payload and the cart are rigidly connected and the mass of the connector is neglected. Second, the system states and its derivatives are measured by using sensors. Third, the cart mass and the rod length are exactly known. Fourth, friction at the ball joint between the payload and the cart is ignored, and this joint is never rotated relative to the link rod. Finally, the angle  $\theta$  is bounded within  $[-\pi, \pi]$ .

## 3. Controller design

## 3.1 Feedback linearization

We approximate the complicated system model in (1) by feedback linearization method [11]. First, we rewrite (1) to

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = f$$
(5)

with

$$q = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} & \dot{\phi} \end{bmatrix}^T, \quad f = \begin{bmatrix} f_x & f_y & 0 & 0 \end{bmatrix}^T$$

A control input vector f in (5) is defined as

$$f = M(q)u + V(q, \dot{q})\dot{q} + G(q)$$
(6)

where a new control vector  $u \in R^4$  is given by

$$u = K_p e(t) + K_d \dot{e}(t) \tag{7}$$

with

$$K_{p} = diag \left\{ k_{p_{x}}, k_{p_{y}}, k_{p_{\theta}}, k_{p_{\theta}} \right\}$$

$$K_{d} = diag \left\{ k_{d_{x}}, k_{d_{y}}, k_{d_{\theta}}, k_{d_{\theta}} \right\}$$

$$e(t) = \left[ e_{x}, e_{y}, e_{\theta}, e_{\phi} \right]^{T} = \left[ x - r_{x}, y - r_{y}, \theta, \phi \right]^{T}$$

$$\dot{e}(t) = \left[ \dot{e}_{x}, \dot{e}_{y}, \dot{e}_{\theta}, \dot{e}_{\phi} \right]^{T} = \left[ \dot{x}, \dot{y}, \dot{\theta}, \dot{\phi} \right]^{T}$$

Substituting (6) to (5), the system model becomes

$$\ddot{q} + K_d \dot{q} + K_p q - K_p r = 0 \tag{8}$$

where a reference vector  $r = \begin{bmatrix} r_x r_y & 0 \end{bmatrix}^T$ . We

properly determine control parameters  $K_p$  and  $K_d$  in (8) according to performance specification through linear system theory. This nominal parameter is optimally performed under which the system model is exactly known. However, this assumption is rarely realistic due to system perturbation, which obviously causes control deviation in practical implementation. Thus, for overcoming such problems, an advanced control scheme is significantly required.

## 3.2 Perturbed system model

We consider a perturbed crane model as

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + \Delta(q,\dot{q}) = f$$
(9)

where  $\Delta(q, \dot{q})$  is additive perturbation term. For compensating system error due to this perturbation, we construct corrective control as an auxiliary input in the system model in (9). Therefore, control input in (7) is linearly composed of nominal input  $u^*(t)$  and corrective input  $\Delta u(t)$ , i.e.,

$$u(t) = u^*(t) + \Delta u(t) \tag{10}$$

Substituting (10) to (9), we similarly obtain

$$\ddot{q} + K_d \dot{q} + K_p q - K_p r + M^{-1} \Delta(q, \dot{q}) - \Delta u = 0$$
 (11)

Intuitively, if  $\Delta u = M^{-1}\Delta(q,\dot{q})$  in (11), the system model is identical to the linear model in (8). In practice, it is hard to mathematically express system perturbation in design procedure, but we assume to know its upper and lower bounds in this paper. This information is even significantly utilized for constructing robust control against perturbation.

## 3.3 Model reference based control design

We use a model reference based adaptive control scheme to derive the corrective control law in (10) for a perturbed crane system. This control design is aimed to match dynamics of an actual system to a reference system model defined as

$$\dot{F}(t) = A_m F(t) + B_m r(t) \tag{12}$$

where reference state matrix  $A_m$  and input matrix  $B_m$ are easily settled based on nominal parameter matrices  $K_p$  and  $K_d$  as

$$A_m = \begin{bmatrix} 0_{4\times4} & I_{4\times4} \\ -K_p & -K_d \end{bmatrix}, \quad B_m = \begin{bmatrix} 0_{4\times4} & K_p \end{bmatrix}^T$$
(13)

Alternatively, we rewrite the perturbation model in (11) as state space equation:

$$\dot{Q}(t) = AQ(t) + Br(t) + \Theta \tag{14}$$

where

$$A = A_m, \quad B = B_m, \quad \Theta = \begin{bmatrix} 0 \\ M^{-1} \Delta(q_1, q_2) - \Delta u \end{bmatrix}$$

Again, this design objective is to establish a corrective control  $\Delta u$  for minimizing dynamic error between two models in (12) and (14), defined as

$$\zeta(t) = F(t) - Q(t) \tag{15}$$

We use Lyapunov stability theory in this design procedure and define a Lyapunov function based on (15) as

$$V(\zeta(t)) = \zeta^{\mathrm{T}}(t)P\zeta(t) \tag{16}$$

where P is a positive definite matrix. The derivative of the Lyapunov function is calculated as

$$\dot{V}(\zeta(t)) = \dot{\zeta}^{\mathrm{T}}(t)P\zeta(t) + \zeta^{\mathrm{T}}(t)P\dot{\zeta}(t)$$
(17)

where

$$\dot{\zeta}(t) = \dot{F}(t) - \dot{Q}(t)$$

$$= A_m F(t) + B_m r(t) - Aq(t) - Br(t) + \Theta \qquad (18)$$

$$= A_m \zeta(t) + A_m q(t) - Aq(t) - Br(t) + \Theta$$

By substituting the third term of (18) to (17), we have

$$\dot{V} = \begin{bmatrix} \zeta^T A_m^T - q^T A^T - r^T B^T + \Theta^T \end{bmatrix} P\zeta + \zeta^T P \begin{bmatrix} A_m \zeta + A_m q - Aq - Br + \Theta \end{bmatrix}$$
(19)
$$= \zeta^T (A_m^T P + P A_m)\zeta + 2\eta$$

where  $\eta = \zeta^T P \Theta$ . According to Lyapunov stability theory,  $\dot{V}$  should be negative, for which the error in (15) is converged. Thus, we have two sufficient sta-

bility conditions for  $\dot{V} < 0$  as follows:

(1) A Lyapunov equation  $A_m^T P + PA_m = -I$  should be negative definite, where *I* is an identity matrix with same size to  $A_m$  and *P*.

(2) A scalar  $\eta$  in (19) should be non-positive, i.e.,  $\eta < 0$ .

We simply seek the first condition (1) by selecting a positive definite matrix P since  $A_m$  is properly given as a Hurwitz type. Next, we satisfy the second condition (2) by analytically settling  $\Delta u$  such that the corrective control is derived from this procedure. Letting

$$\boldsymbol{\zeta} = \begin{bmatrix} \zeta_1 & \zeta_2 \end{bmatrix}^T, \quad \boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

the second condition is expanded as

$$\eta = \left(\zeta_1 p_{12} + \zeta_2 p_{22}\right) \left[ M^{-1} \Delta(q_1, q_2) - \Delta u \right] < 0 \quad (20)$$

Since the elements of matrix M are given and its inverse matrix  $M^1$  is simply calculated in design procedure, we obviously define its bounds. As well, we assume that the bound of perturbation  $\Delta(q_1,q_2)$  in (20) is known. Using these facts, we easily derive  $\Delta u(t)$  to satisfy an inequality in (20) as

$$\Delta u = sign\left(\zeta_1 p_{12} + \zeta_2 p_{22}\right) \max\left(\left|M^{-1}\right|\right) \max\left(\left|\Delta(q_1, q_2)\right|\right)$$
(21)

This control law is simply constructed based on the maximum bounds, which are actually fixed in practice. But the sign of  $(\zeta_1 p_{12} + \zeta_2 p_{22})$  in (21) is changed according to modeling errors  $\zeta_1$  and  $\zeta_2$  generated due to perturbation. This realizes accomplishing the adaptive control scheme.

## 4. Stability analysis of perturbed crane systems

We carry out stability analysis of the crane control system by using Lyapunov perturbation stability theory for non-vanishing systems.

## Theorem 1 (converse theorem)

Consider a nonlinear dynamic system with an equilibrium point x = 0 as

$$\dot{x} = f(x,t) \tag{22}$$

where norm of  $x \in \mathbb{R}^n$  is less than a positive constant r,

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i.e., ||x|| < r. If a solution of this system is expressed as

$$\|x(t)\| \le k \|x(t_0)\| e^{-\lambda(t-t_0)}, \quad 0 \le t_0 \le t$$
 (23)

where k > 0, a Lyapunov function satisfies the following three inequalities:

$$c_1 \|x\|^2 \le V(x,t) \le c_2 \|x\|^2$$
 (24)

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) \le -c_3 \left\| x \right\|^2$$
(25)

$$\left\|\frac{\partial V}{\partial x}\right\| \le c_4 \left\|x\right\| \tag{26}$$

where  $c_1, c_2, c_3, c_4 > 0$ .

Consider a nonlinear dynamic model with nominal and perturbed terms, respectively, as

$$\dot{x} = f(x,t) + g(x,t) \tag{27}$$

where f(x,t) is a nominal system function and g(x,t) is a perturbation function. Assume that a perturbation function is bounded as

$$\left\|g(x,t)\right\| \le \gamma \left\|x(t)\right\| \tag{28}$$

where  $\gamma > 0$ . For a perturbed model, we have a derivative function as

$$\dot{V}(x,t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f(x,t) + \frac{\partial V}{\partial x}g(x,t)$$
(29)

where two partial differential terms  $\partial V/\partial t$  and  $(\partial V/\partial x)f(x,t)$  involve a nominal function, which are simply calculated from given system equation. A partial differential term  $(\partial V/\partial x)g(x,t)$  with respect to a perturbation function is rarely obtained from analytical calculus since information about perturbation is unknown. However, by using the inequalities in (25) and (28), we have

$$\dot{V}(x,t) \leq -c_3 \left\| x \right\|^2 + \left\| \frac{\partial V}{\partial x} \right\| \left\| g(x,t) \right\|$$

$$\leq -c_3 \left\| x \right\|^2 + c_4 \gamma \left\| x \right\|^2$$
(30)

If  $\gamma$  is very small, i.e.,  $\gamma < c_3 / c_4$ , the inequality in (30) is rewritten by

$$\dot{V}(t,x) \le -(c_3 - \gamma c_4) \|x\|^2$$
 (31)

From this result, we conclude a Lyapunov stability criterion for perturbation systems.

# Theorem 2 (Lyapunov perturbation stability)

For a nominal function f(x, t) in (27) which is satisfied with relation to (24)-(26), if a perturbation function g(x, t) is concerned with (30) and (31), a perturbed system in (27) is asymptotically stable at an equilibrium point x = 0.

If a nominal model in (27) is linear, i.e., f(x, t) = Ax(t), a perturbed system model is expressed as

$$\dot{x}(t) = Ax(t) + g(x,t) \tag{32}$$

with real  $\{\lambda(A)\} < 0$  where  $\lambda$  is eigenvalue. This statement indicates that if a nominal system is asymptotically stable, solution of a square matrix *P* exists from Lyapunov equation

$$PA + A^{\mathrm{T}}P = -Q \tag{33}$$

where a matrix Q is positive definite. Using eigenvalues of a matrix P, for Lyapunov function  $V(x) = x^T P x$ , we have

$$\lambda_{\min}(P) \|x\|_2^2 \le V(x) \le \lambda_{\max}(P) \|x\|_2^2 \tag{34}$$

$$\frac{\partial V}{\partial x}Ax = -x^{\mathrm{T}}Qx \le -\lambda_{\min}(P) \|x\|_{2}^{2}$$
(35)

$$\frac{\left\|\frac{\partial V}{\partial x}\right\|_{2}}{\left\|=\left\|2x^{\mathrm{T}}P\right\|_{2} \le 2\left\|P\right\|_{2}\left\|x\right\|_{2}}$$

$$= 2\lambda_{\mathrm{max}}(P)\left\|x\right\|_{2}$$
(36)

Thus, its derivative for the perturbation term is obtained as

$$\dot{V}(x) = -\lambda_{\min}(Q) \|x\|_2^2 + 2\lambda_{\max}(P)\gamma \|x\|_2^2$$
 (37)

From (37) we have a sufficient condition for  $\dot{V}(x) < 0$  as

$$\gamma < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \tag{38}$$

We use this result for the stability criterion of our crane control system including perturbation.

To utilize the procedure, we separately express nominal and perturbation terms in (11) as

$$f(q) = Aq(t) = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix}$$
(39)

$$g(q) = \begin{bmatrix} 0\\ K_p r + M^{-1} \Delta + \Delta u \end{bmatrix}$$
(40)

We simply realize stable dynamics for a nominal function f(x) in (39) by determining proper values of  $K_p$  and  $K_d$  by means of linear system theory. Letting  $Q = I_{8\times8}$  in (33) for simplicity and using a matrix A in (39), for satisfying a Lyapunov equation in (33), we thus obtain

$$P = \frac{1}{2} \begin{bmatrix} K_d K_p^{-1} + K_d^{-1} & -K_p^{-1} - 2(K_d^{-1})^2 \\ K_p^{-1}(K_d^{-1})^2 & K_d^{-1}K_p^{-1} \end{bmatrix}$$
(41)

Since  $K_p, K_d \in \mathbb{R}^{4 \times 4}$ , there are eight eigenvalues in a matrix *P*. We denote a maximum eigenvalue of it as

$$\lambda^* = \max\left\{\lambda_{p_1}, \lambda_{p_2}, \dots, \lambda_{p_8}\right\}$$
(42)

According to (34)-(36), we obtain  $c_3 = I$  and  $c_4 = \lambda^*$  in (39) and (40); thus the norm of a perturbation function g(q) in (40) is expanded as

$$\begin{aligned} \left\| g(q) \right\|_{2} &= \left| K_{p}r + M^{-1}\Delta(q,\dot{q}) + \Delta u \right| \\ &\leq \left| K_{p}r \right| + \left| M^{-1}\Delta(q,\dot{q}) + \Delta u \right| \\ &\leq \left| K_{p}r \right| + \left\| M^{-1} \right\| \left| \Delta(q,\dot{q}) \right| + \left| \max(\Delta(u)) \right| \end{aligned}$$
(43)

Alternatively, we express a perturbation term in (40) with constant  $\gamma$  as

$$\Delta(q_1, q_2) = \gamma \delta(q_1, q_2) \tag{44}$$

In general, the perturbation norm is relatively smaller than one of a state vector Q, i.e.,

$$\left\|\delta(q_1, q_2)\right\|_2 \le \left\|Q\right\|_2 \tag{45}$$

Therefore, the derivative of the Lyapunov function for our system model becomes

$$\dot{V}(q) = -\|Q\|_{2}^{2} + 2\lambda^{*} \left\{ \left| K_{p}r \right| + \|M^{-1}\|\gamma\|Q\|_{2}^{2} + \max(\Delta u) \right\}^{(46)}$$

From this inequality, we simply seek a sufficient condition for stability as

$$\gamma < -\frac{\left|K_{p}r\right| + \max(\Delta u)}{\left\|M^{-1}\right\|\max(\left\|q\right\|_{2}^{2})}$$
(47)

#### Remark

The proposed crane control system with perturbation is asymptotically stable if there is a constant value satisfied with an inequality in (47) for system dynamics.

## 5. Simulation study

The proposed crane control system is numerically simulated to evaluate its control performance. We carry out three simulation experiments in turn: First, we design nominal PD control for a non-perturbed crane model, where a proper parameter vector is analytically selected by using linear system theory. Second, we apply a constructed PD control to a perturbed crane system, and lastly, we design and numerically test the proposed control for a perturbed crane system.

**Case 1**: We adopt parameter values of the crane system from [5] as follows:  $m_p = 160$  [kg],  $m_c = 23$  [kg],  $m_r=190$ [kg], L=2.5[m], and I=1.5[kg·m<sup>2</sup>]. We settle reference values  $r_x=10$ [m] and  $r_y=3$ [m], respectively, and control time interval is 80 sec. Performance specification for controller design is given as follows: zero steady-state error, no overshoot behavior, and 30 second settling time. For this specification, we analytically determine nominal PD parameter values as

$$K_p = diag\{0.1, 0.25, 0.5, 0.35\}$$
  
 $K_d = diag\{0.35, 0.5, 0.8, 0.75\}$ 

We apply this nominal PD control for the nonperturbed crane model and plot its system response shown in Fig. 2. From this result, we observe that there is no overshoot in the position responses for both x- and y-axis, and their settling times are observed at about 12 and 15 sec. As well, trajectories of two angles shown in Figs. 2(c) and (d) are obviously converged into an equilibrium state within the given control time. As expected, the nominal PD control is suitably performed against the crane system avoiding perturbation. In other words, the PD control properly works for the nominal crane system.

Case 2: We test the PD control designed in Case 1 by applying it to a perturbed crane model. To realize system perturbation, we increase the mass of the payload to 3000 [kg]. Namely, the change of payload mass in crane systems is reasonably encountered in industrial fields. The rest of the simulation environment is identical to Case 1. We plot system response for this control system in Fig. 3. Apparently, the responses are not converged to an equilibrium state within the given time interval, unlike Case 1. The crane positions hardly reach the reference values, and trajectories of the angles are irregularly oscillating. Thus, the control dynamics is wholly unstable. The simulation result demonstrates that nominal PD control is unsatisfactorily accomplished for the perturbed crane system.

Case 3: This simulation is aimed to evaluate our proposed control approach for the perturbed crane system configured in Case 2. We use the PD control constructed in Case 1 as a nominal control and design a corrective control based on the guideline stated in Section 3. Information of the crane perturbation provided in Case 2 is used to define its maximum bound for control design. Fig. 4 shows system responses for the proposed control system. All of the trajectories are obviously converged to their reference values satisfying the performance specifications. Obviously, there is no overshoot in transient response, and settling time for two positions is about 25 and 28 sec, respectively; these are more increased than those of Case 1, because the mass of the payload is much bigger. We observe that this control performance is superior to that of Case 2 and conclude that the proposed control is effectively carried out against the perturbed system.

## 6. Conclusion

This paper presents an adaptive control approach for complicated nonlinear crane systems with perturbation. The proposed control is linearly composed of nominal PD and corrective control frameworks. The nominal PD control is constructed by using a nonperturbed crane model that is approximated through a feedback linearization technique. The corrective con-



Fig. 2. Crane system responses (Case 1).



Fig. 3. Crane system responses (Case 2).



Fig. 4. Crane system responses (Case 3).

trol is established from the perturbed crane model by means of Lyapunov stability theory to compensate system error occurring due to uncertain perturbation. In addition, we conduct stability analysis for the perturbation crane system embedded with the proposed control. We analytically prove the implemented crane system is asymptotically stable with respect to bound of the perturbation. For evaluation of the proposed control scheme, numerical simulation is carried out by comparing the traditional control method. We realize system perturbation by changing the mass of the payload within an allowed bound. The simulation result demonstrates its superiority to the PD control well-used in industrial crane control. Future work will include real-time experiments to examine its practicality.

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